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SUMMARY

An experimental investigation was conducted to determine the viscous slosh-damping characteristics of several liquids having a wide range of kinematic viscosities. The tests were conducted with three rigid spherical tanks 9.5, 20.5, and 32.0 inches in diameter. The kinematic viscosity of the contained liquid was varied from 1.23×10^{-6} to 1.183×10^{-2} square foot per second by utilizing mercury, acetylene tetrabromide, and several mixtures of water and glycerine. The first-mode slosh-force parameter was generalized for all values of kinematic viscosity and tank diameter investigated when presented as a function of dimensionless viscosity and excitation-amplitude parameters. The average first-mode damping ratio was observed to be independent of the excitation amplitude over the range investigated and was generalized for all values of kinematic viscosity and tank diameter when presented as a function of the dimensionless viscosity parameter only.

INTRODUCTION

One of the problem areas currently associated with liquid-fuel propulsion systems used in missiles and space-flight vehicles is that of damping the oscillations of the propellant masses that may result from the dynamic response of the vehicle to the attitude-stabilization-control system. The oscillation of the propellants, when allowed to continue undamped, can create slosh forces that may have an adverse effect on the stability and structural integrity of the vehicle. When liquid sloshing occurs in a spherical tank, for example, the maximum slosh forces approach a value equal to approximately one-fourth of the apparent weight of the contained liquid (ref. 1). Generally, the most effective method of decreasing the slosh forces is to reduce the amplitude of the liquid oscillations by increasing the damping applied to the contained liquid.

Several methods of increasing the damping applied to the contained liquid have been investigated for several different tank configurations. Among these are the use of floating cans and asymmetrical or annular ring baffles (refs. 2 to 7) and the use of positive-expulsion bags and diaphragms (refs. 8 and 9). Another method of increasing the liquid damping that may warrant consideration is the use of the viscous damping inherent in high viscosity liquids or, as applied to missiles or space vehicles, the use of thixotropic propellants. Knowledge of the viscous-damping characteristics of liquid sloshing would enable prediction of maximum slosh forces and damping ratios for thixotropic propellants as well as for more conventional propellants of varying viscosity.

Accordingly, an experimental investigation was conducted at the NASA Lewis Research Center to study the potential slosh-damping effectiveness of viscous liquids having a range of kinematic viscosities from 1.23×10^{-6} to 1.183×10^{-2} square foot per second. The investigation was conducted utilizing three rigid spherical tanks 9.5, 20.5, and 32.0 inches in diameter. The spherical tank configuration was chosen because (1) the minimum tank weight for a given volume provides definite advantages for some space-vehicle applications and (2) the information presently available on liquid sloshing in spherical tank configurations is relatively scarce. The effects of liquid kinematic viscosity, as well as excitation amplitude and frequency, on the horizontal slosh forces and damping ratios were investigated.

SYMBOLS

B	viscosity parameter, $\nu/\sqrt{gD^3}$, dimensionless
D	tank diameter, ft
F_s	horizontal slosh force, lb
g	vertical acceleration, 32.174 ft/sec^2
h	liquid depth, ft
$h/2R$	liquid depth ratio, dimensionless
R	tank radius, ft
X_o	excitation amplitude, ft
X_o/D	excitation-amplitude parameter, dimensionless
α	oscillatory excitation frequency, radians/sec
δ	damping ratio, $\ln\left[(F_s)_n/(F_s)_{n+1}\right]$
ρ	liquid mass density, slugs/cu ft
η	excitation-frequency parameter, $\alpha\sqrt{R/g}$, dimensionless

λ slosh-force parameter, $F_s/\rho g D^3$, dimensionless

ν liquid kinematic viscosity, ft^2/sec

Subscript:

n cycle number (1,2,3,4, . . .,n)

APPARATUS AND INSTRUMENTATION

Two experimental test facilities were utilized in this investigation. The smaller experimental test facility shown in figure 1 was identical to that used in the investigations of liquid sloshing reported in references 1, 8, and 9. A 9.5-inch-diameter spherical tank was formed in a laminated lucite block. When placed on the test facility, the tank was mounted on four ball bearings and was free to oscillate in one direction in the horizontal plane. The sinusoidal motion of the tank was produced by a slider-crank mechanism driven through a variable-speed transmission by an a-c electric motor. The excitation frequency could be varied from 0 to 5 cycles per second. The electric driving motor was wired so that the alternating current could be removed from the field and a direct current could be applied to one of the field windings. This enabled the oscillatory motion of the tank to be "quick-stopped" so that only the residual horizontal forces resulting from the liquid motion could be measured. These residual slosh forces were sensed by a strain gage mounted between the tank and the slider-crank mechanism. The signal from the strain gage was displayed by means of a continuously recording strip chart.

The larger experimental test facility is shown in figure 2. Spherical tanks with diameters of 20.5 and 32.0 inches were mounted on a test bed that was suspended from a frame through three vertically oriented load cells and one horizontally oriented load cell. The frame was suspended from overhead crossbeams and was free to oscillate in one direction in the horizontal plane. The driving force was provided by a hydraulic cylinder and piston. The excitation amplitude could be varied from 0 to 1 inch, and the excitation frequency could be varied from 0 to 20 cycles per second. A sinusoidal excitation wave form was used for this investigation. The electric and hydraulic control circuits for the driving mechanism were designed to enable the oscillatory motion of the frame, test bed, and tank to be quick-stopped at a point of zero velocity during any given cycle of oscillation so that only the residual horizontal forces resulting from the liquid motion could be measured. The horizontal slosh forces were sensed by the horizontal load cell, and the signal was displayed on a continuously recording strip chart. The three vertical load cells were not utilized in the present investigation for instrumentation.

The slosh liquids used in this investigation were water ($\nu = 1.062 \times 10^{-5}$ ft^2/sec), glycerine ($\nu = 1.183 \times 10^{-2}$ ft^2/sec), and a mixture of these two liquids in varying percentages to obtain intermediate values of kinematic viscosity. Unpublished NASA data also included for this investigation were damping ratios determined from experimental tests utilizing mercury ($\nu = 1.23 \times 10^{-6}$ ft^2/sec) and acetylene tetrabromide, commonly known as TBE ($\nu = 3.538 \times 10^{-5}$ ft^2/sec).

PROCEDURE

The effectiveness of slosh-suppression methods is the most critical to missile design at conditions where the slosh forces are a maximum. From an investigation of unrestricted sloshing characteristics (ref. 1), it was observed that the maximum slosh forces occurred at the first natural mode and at a liquid-depth ratio ($h/2R$) of 0.50. Therefore, all of the tests reported herein were conducted at a liquid-depth ratio of 0.50.

For each data point taken on either test facility, the tank was oscillated at a preselected excitation frequency and amplitude until the wave height of the liquid on the tank wall had reached the maximum value that could be obtained without the surface of the liquid breaking up and showering liquid throughout the tank. The oscillatory motion of the tank was then quick-stopped, and the residual horizontal slosh forces were recorded. The excitation amplitude was held constant at 0.100 inch while excitation frequencies encompassing the fundamental mode were investigated with the smaller test facility. Because of the limitations of this facility, an excitation amplitude smaller than 0.100 inch would not permit accurate slosh-force measurements of the higher viscosity liquids, while a larger excitation amplitude forced the wave forms of the lower viscosity liquids to build up so rapidly that the oscillatory motion of the tank could not be quick-stopped at the proper time. Excitation amplitudes from 0.050 to 0.900 inch and excitation frequencies encompassing the first two natural modes of oscillation were investigated with the larger test facility.

DATA REDUCTION

The experimental values of the slosh forces and excitation frequencies were reduced to the dimensionless parameters $\lambda = F_s/\rho g D^3$ and $\eta = \alpha\sqrt{R/g}$, respectively. The slosh-force parameter was calculated by using the magnitude of the first force peak occurring immediately after the oscillatory motion of the tank had been quick-stopped. The damping ratios were calculated as the logarithmic decrement of a smooth curve faired through successive force peaks. The g term appearing in the parameters is equal to the vertical acceleration of the tank (32.174 ft/sec^2 for the present investigation).

PRESENTATION AND DISCUSSION OF RESULTS

Slosh-Force Parameter

General effect of kinematic viscosity. - The general slosh-force suppression characteristics obtained by increasing the kinematic viscosity of the contained liquid from that of water ($\nu = 0.1 \times 10^{-4}$ ft^2/sec) to that of 100 percent glycerine ($\nu = 118.3 \times 10^{-4}$ ft^2/sec) in each of the three spherical tanks is presented in figure 3 for a nearly constant value of the excitation-amplitude parameter X_o/D. For each tank diameter, the dimensionless slosh-force parameter increased and reached a maximum value as the excitation-frequency parameter increased and approached that value associated with the first natural mode ($\eta = 1.254$, ref. 1). The force parameter then decreased with a further increase of the excitation-frequency parameter. For each tank oscillated at a given value of the excitation-frequency parameter, the effect of increasing the kinematic viscosity of the contained liquid from 0.1×10^{-4} to 118.3×10^{-4} square foot per second was to reduce the wave height to which the liquid mass could be driven. The consequent reduction of the first-mode slosh-force parameter (the maximum value of the force parameter that occurred when the excitation frequency was equal to the fundamental frequency of oscillation of the contained liquid) was as much as 67 percent in the 9.5-inch-diameter tank, 55 percent in the 20.5-inch-diameter tank, and 28 percent in the 32.0-inch-diameter tank. The small, relatively insignificant force peak shown at the second natural mode ($\eta = 2.322$) was almost completely eliminated by the increased kinematic viscosity of 100 percent glycerine. The value of the excitation-frequency parameter at which the maximum slosh forces occurred remained essentially constant for all tank diameters investigated and was not affected by the kinematic viscosity of the contained slosh liquid.

The slosh-force parameters obtained for a relatively nonviscous liquid (water) reached a maximum value of about 0.06 at the first mode. It was observed that the value of the excitation-amplitude parameter ($X_o/D \approx 0.01$) (1) drove the wave form of the liquid oscillations to approximately the same value of wave-height to tank-diameter ratio for all three tanks and (2) was sufficiently large to drive the wave-height to tank-diameter ratio to nearly the maximum value dictated by the spherical tank configuration and still maintain the nearly flat liquid surface characteristics of first-mode sloshing.

With 100 percent glycerine ($\nu = 118.3 \times 10^{-4}$ ft^2/sec), the first-mode slosh-force parameter increased with tank diameter for nearly constant values of the excitation-amplitude parameter. The effect of the large increase of kinematic viscosity on this force parameter is discussed further in the appendix.

The effect of a variation of the excitation-amplitude parameter (X_o/D) on the slosh-force parameter utilizing 100 percent glycerine as the contained slosh liquid is presented in figure 4(a) for the 20.5-inch-diameter tank and in figure 4(b) for the 32.0-inch-diameter tank. For each tank diameter, an increase of X_o/D resulted in an increase of the wave height to which the liquid surface was driven, which increased the value of the force parameter. The first-mode slosh-force parameter generally increased without regard to tank diameter with an increase in the excitation amplitude X_o.

The effect of kinematic viscosity on the first-mode slosh-force parameter over a wide range of X_o/D is presented for the 32.0-inch-diameter tank in figure 5. The force parameter (1) decreased with an increase of kinematic viscosity and (2) increased with an increase of the excitation amplitude. The higher viscosity of 100 percent glycerine decreased the first-mode slosh-force parameter from that obtained for water by 60 percent or more for values of $X_o/D < 0.004$, for example, but decreased the force parameter by only about 10 percent at a value of X_o/D of 0.0281, which was the maximum value investigated.

The variation of the slosh-force-parameter levels occurring at a given excitation frequency or amplitude is due to experimental technique. The scatter of the data is attributed to the difficulty in visually observing that the wave height of the liquid surface had reached a maximum value and in quick-stopping the oscillatory motion of the tank during the same cycle.

A phenomenon known as liquid swirl, which is frequently encountered in liquid sloshing, occurs most often at excitation frequencies near or equal to the fundamental frequencies. Although frequently observed with lower viscosity liquids, liquid swirl was never observed for those liquids having kinematic viscosities greater than approximately 10×10^{-4} square foot per second.

Generalized results. - The first-mode slosh-force parameters obtained for water tended to become generalized (within experimental accuracy) relative to tank diameter when presented as a function of the excitation-amplitude parameter X_o/D, as shown in figure 6. The force parameter increased to a maximum value of 0.061 at an X_o/D of 0.0125 and remained constant with any further increase of X_o/D. At $X_o/D = 0.0125$, the wave height of the surface of the liquid at the tank wall became limited by the physical configuration of the tank and was, therefore, the maximum that could be obtained. The only effect of increasing the value of X_o/D beyond that of 0.0125 was to breakup the relatively flat liquid surface causing a small portion of the liquid to splash over the top and shower down through the center of the tank (fig. 2(b)). However, the first-mode force parameters obtained for 100 percent glycerine did not generalize for various tank diameters when presented as a function of X_o/D. At a given value of X_o/D, the force parameters for 100 percent glycerine decreased with the tank diameter below those values obtained with water for the reasons discussed in the appendix. It appears that the first-mode slosh-force parameters may closely approach those values obtained for water with either very large values of tank diameter or excitation amplitude.

A dimensionless viscosity parameter $B = \nu/\sqrt{gD^3}$, which is similar to that derived in reference 10, was applied to the experimental data in an effort to predict more readily the first-mode slosh forces over the complete range of liquid kinematic viscosities. The first-mode slosh-force parameters obtained for all tank diameters investigated are presented as a function of the viscosity parameter for a value of X_o/D of about 0.01 in figure 7. The force parameter was generalized relative to tank diameter and liquid kinematic viscosity and decreased with an increase of the viscosity parameter.

To predict the values of the first-mode slosh-force parameter over a wide range of X_o/D, the experimental data obtained from the 32.0-inch-diameter tank (fig. 5) are presented as a function of the viscosity parameter in figure 8. With the assumption that the viscosity parameter does generalize the data for values of X_o/D other than those presented in figure 7, the family of curves generated in figure 8 is applicable in predicting the first-mode slosh forces that could be expected to occur for any value of the viscosity parameter within the range investigated and for a tank of any diameter oscillated over a range of X_o/D from 0.00156 to 0.02810. To obtain substantial reductions of the first-mode slosh-force parameter, the viscosity parameter must be high (maximum liquid kinematic viscosity and minimum tank diameter), and X_o/D should be a minimum.

Damping Ratio

General effect of kinematic viscosity. - The damping ratios (or logarithmic decrements) obtained over a range of liquid kinematic viscosities are presented as a function of the excitation-frequency parameter in figures 9(a) to (c) for tank diameters of 9.5, 20.5, and 32.0 inches, respectively. In general, the damping ratios obtained for each tank diameter increased with the kinematic viscosity of the contained liquid and were essentially independent of the excitation amplitudes investigated. The damping ratios appeared to be dependent upon the excitation-frequency parameter when both the wave height and tank diameter were small. However, the damping ratios obtained in the 32.0-inch-diameter tank were, within experimental accuracy, independent of the excitation-frequency parameter.

Figure 10 presents the average first-mode damping ratio (an average of damping ratios obtained for excitation frequencies at and near that of the fundamental mode) as a function of the liquid kinematic viscosity for the three tank diameters investigated. The damping ratio increased with kinematic viscosity for all the tank diameters. However, for a constant value of the kinematic viscosity, the damping ratios decreased as the tank diameter increased; this decrease is a direct result of the reduction of the wetted-tank-wall-area to volume ratio discussed in the appendix. The error bars on the curve for the 32.0-inch-diameter tank indicate the range of damping ratios (data scatter) obtained for values of X_o from 0.050 to 0.900 inch. This data scatter for the damping ratios presented in figures 9 and 10 is due to (1) the limitations of the test facility and (2) the difficulty in fairing a curve through successive force peaks, which would provide a constant value of the damping ratio through successive cycles of the liquid oscillations.

Generalized results. - The average first-mode damping ratios obtained for all liquids investigated (TBE, mercury, and mixtures of glycerine and water) are presented in figure 11 as a function of the dimensionless viscosity parameter for the range of tank diameters investigated. The viscosity parameter did generalize the effect of kinematic viscosity and tank diameter on the first-mode damping ratio for spherical tanks at a liquid-depth ratio (h/2R) of 0.50. The fact that a straight line may be faired through experimental data indicates that a relation exists between the first-mode damping ratio and the viscosity parameter in the form

$$\delta = c\left(\frac{\nu \times 10^4}{\sqrt{gD^3}}\right)^N \tag{1}$$

where c and N are constants that may be determined from the curve. The constants were found to have the values $c = 0.131$ and $N = 0.359$. The relation between the first-mode damping ratio and the viscosity parameter is then

$$\delta = 0.131\left(\frac{\nu \times 10^4}{\sqrt{gD^3}}\right)^{0.359} \tag{2}$$

SUMMARY OF RESULTS

An experimental investigation was conducted utilizing three spherical tanks 9.5, 20.5, and 32.0 inches in diameter to determine the effect of a large variation of liquid kinematic viscosity on the horizontal slosh forces over a range of (1) excitation frequencies encompassing the first and second natural modes of oscillation of the contained liquid and (2) excitation amplitudes from 0.050 to 0.900 inch. The liquid-depth ratio (h/2R) was maintained at a value of 0.50 where the maximum first-mode slosh forces have been shown to occur.

The slosh forces reached a maximum value at excitation frequencies equal to the first natural mode of oscillation of the contained liquid. A smaller, relatively insignificant slosh force peak occurred at the second natural mode frequency. An increase of the viscosity of the contained liquid reduced the height to which the first- and second-mode wave forms could be driven, which reduced the first- and second-mode slosh forces. The natural frequencies of oscillation of the first and second mode were independent of the liquid viscosity.

The first-mode slosh-force parameters were generalized within experimental accuracy for all values of liquid kinematic viscosity and tank diameter investigated when presented as a function of a dimensionless viscosity parameter $\nu/\sqrt{gD^3}$ and the excitation-amplitude parameter X_o/D. The effectiveness of the higher viscosity liquids in reducing the first-mode slosh forces decreased as the tank diameter and excitation amplitude increased.

The average first-mode damping ratios were observed to be independent of the excitation amplitude and increased with (1) an increase of the liquid kinematic viscosity or (2) a decrease of tank diameter. The viscosity parameter generalized the effect of kinematic viscosity and tank diameter on the average first-mode damping ratios that could then be expressed by the equation

$$\delta = 0.131 \left(\frac{\nu \times 10^4}{\sqrt{gD^3}}\right)^{0.359}$$

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, July 11, 1963

APPENDIX - EFFECT OF A WETTED-TANK-WALL AREA TO VOLUME RATIO ON THE FIRST-MODE SLOSH-FORCE PARAMETER AND DAMPING RATIO

Slosh Forces

The first-mode slosh forces produced by the liquid oscillation in a half-full spherical tank are dependent upon (1) tank diameter, (2) liquid mass density, (3) gravity field under which the tank is oscillated, and (4) liquid-wave-height to tank-diameter ratio. The dimensionless slosh-force parameter generalizes the effect of variables (1) to (3) for constant values of (4). The wave-height to tank-diameter ratio is primarily dependent upon (1) the excitation-amplitude parameter at which the tank and its contained liquid are oscillated and (2) the liquid viscosity.

The wave-height to tank-diameter ratio for a given excitation-amplitude parameter X_o/D would be expected to be determined by the ratio of the total viscous friction forces to the slosh forces. The viscous friction forces, which damp the liquid oscillations, are proportional to the wetted surface area of the tank wall, while the slosh forces are proportional to the tank volume. The ratio of the viscous friction forces to the first-mode slosh forces for a given liquid can then be given by the equation

$$\frac{F_f}{F_s} = K \frac{A}{V} = K \frac{1/2\pi D^2}{1/6\pi D^3} = 3KD^{-1} \tag{A1}$$

where

F_f total viscous friction forces

K constant of proportionality

A wetted tank-wall area

V tank volume

For given values of liquid mass density and excitation amplitude, the value of K is related to the liquid viscosity and increases with an increase in viscosity.

When considering a relatively nonviscous liquid (water, e.g.) sinusoidally excited at the first natural mode frequency, it was observed that the wave-height to tank-diameter ratio, and, therefore, the first-mode slosh-force parameter, remained essentially constant with a variation of tank diameter when the excitation-amplitude parameter X_o/D remained constant. In the case of a relatively nonviscous liquid, the value of K would be quite small; therefore, the value of the viscous friction forces would remain small as the tank diameter increased, and a variation of the tank diameter would be expected to produce only a slight variation in the value of the first-mode slosh-force parameter.

For a relatively viscous liquid (100 percent glycerine, e.g.) the value of K would be quite large, and the viscous friction forces, therefore, would be large and would vary widely as the tank diameter was varied. The viscous friction forces would also become more significant when compared to the slosh forces as the tank diameter decreased. For constant values of X_o/D, the wave-height to tank-diameter ratio and, therefore, the first-mode slosh-force parameter decreased as the tank diameter decreased. It was necessary to drive the tank at larger values of X_o/D as the diameter decreased to overcome the viscous friction forces and to maintain a constant wave-height to tank-diameter ratio.

Damping Ratio

The damping ratio (logarithmic decrement) is a measure of the time rate of decay of the slosh forces after the disturbance has been removed. Since the instantaneous wetted surface area of a given diameter tank is essentially independent of the wave height of a contained liquid undergoing characteristic first-mode sloshing, the damping ratio would be expected to be independent of the excitation amplitude. The damping ratio would be expected to be affected by the ratio of the viscous friction forces to the slosh forces and would, therefore, increase with either an increase in liquid viscosity or a decrease in tank diameter. Damping ratios obtained from the experimental data are summarized in figure 12 as a function of the A/V ratio.

REFERENCES

1. Stofan, Andrew J., and Amstead, Alfred L.: Analytical and Experimental Investigation of Forces and Frequencies Resulting from Liquid Sloshing in a Spherical Tank. NASA TN D-1281, 1962.

2. Silveira, Milton A., Stephens, David G., and Leonard, H. Wayne: An Experimental Investigation of the Damping of Liquid Oscillations in Cylindrical Tanks with Various Baffles. NASA TN D-715, 1961.

3. Stephens, David G., Leonard, H. Wayne, and Silveira, Milton A.: An Experimental Investigation of the Damping of Liquid Oscillations in an Oblate Spheroidal Tank With and Without Baffles. NASA TN D-808, 1961.

4. Abramson, H. Norman, and Ransleben, Guido E., Jr.: A Note on the Effectiveness of Two Types of Slosh Suppression Devices. Tech. Rep. 6, Southwest Res. Inst., June 15, 1959.

5. Bauer, Helmut F.: Fluid Oscillations in a Cylindrical Tank with Damping. Rep. DA-TR-4-58, Army Ballistic Missile Agency, Apr. 23, 1958.

6. Cole, Henry A., Jr., and Gambucci, Bruno J.: Tests of an Asymmetrical Baffle for Fuel-Sloshing Suppression. NASA TN D-1036, 1961.

7. Abramson, H. Norman, Chu, Wen-Hwa, and Garza, R. Luis: Liquid Sloshing in Spherical Tanks. Tech. Rep. 2, Southwest Res. Inst., Mar. 15, 1962.

8. Stofan, Andrew J., and Pavli, Albert J.: Experimental Damping of Liquid Oscillations in a Spherical Tank by Positive-Expulsion Bags and Diaphragms. NASA TN D-1311, 1962.

9. Stofan, Andrew J., and Sumner, Irving E.: Experimental Investigation of the Slosh-Damping Effectiveness of Positive-Expulsion Bags and Diaphragm in Spherical Tanks. NASA TN D-1712, 1963.

10. Berlot, R. R., Birkhoff, G., and Miles, J. W.: Slosh Damping in a Rigid Cylindrical Tank. Rep. GM-TR-263, Space Tech. Labs., Oct. 9, 1957.

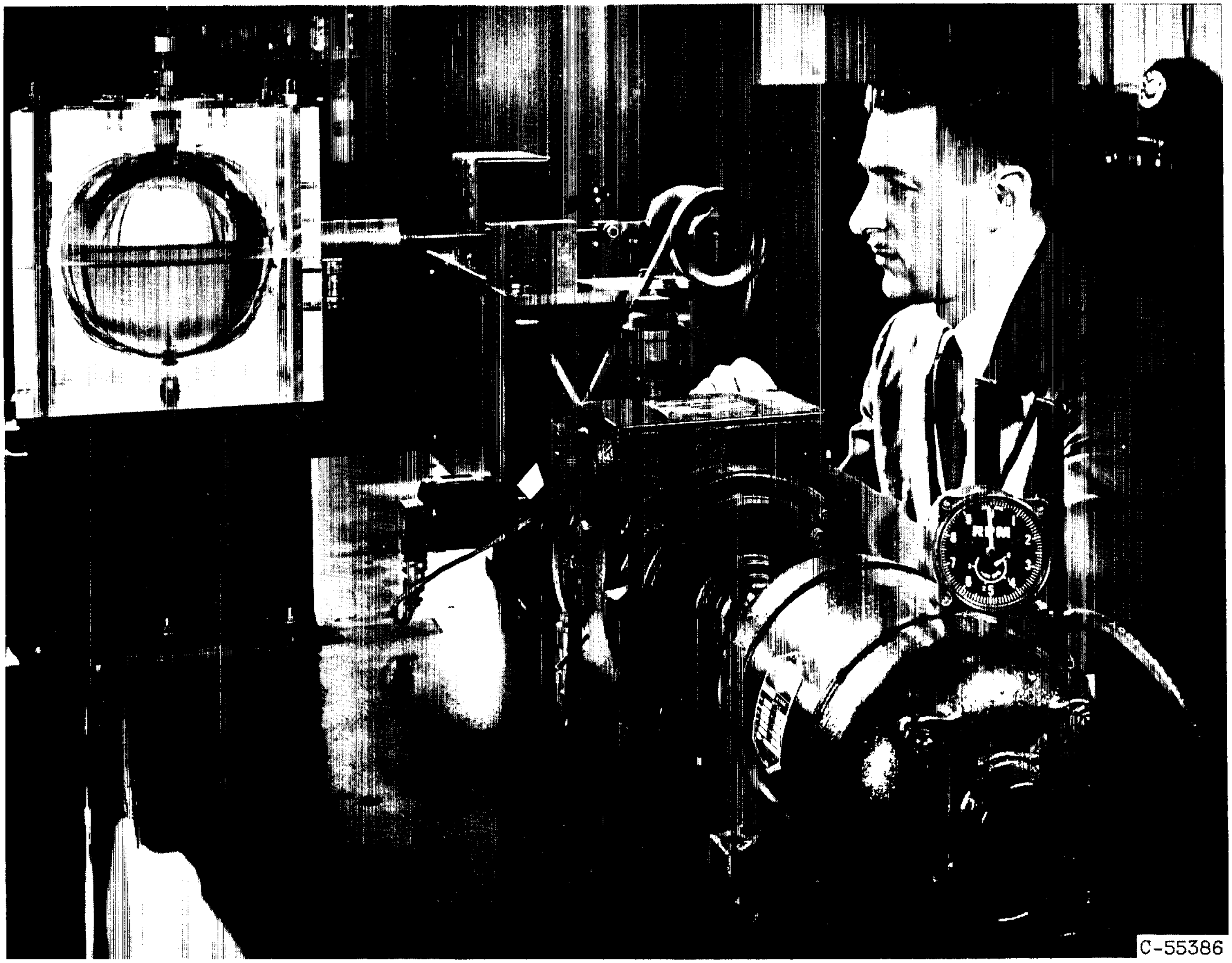
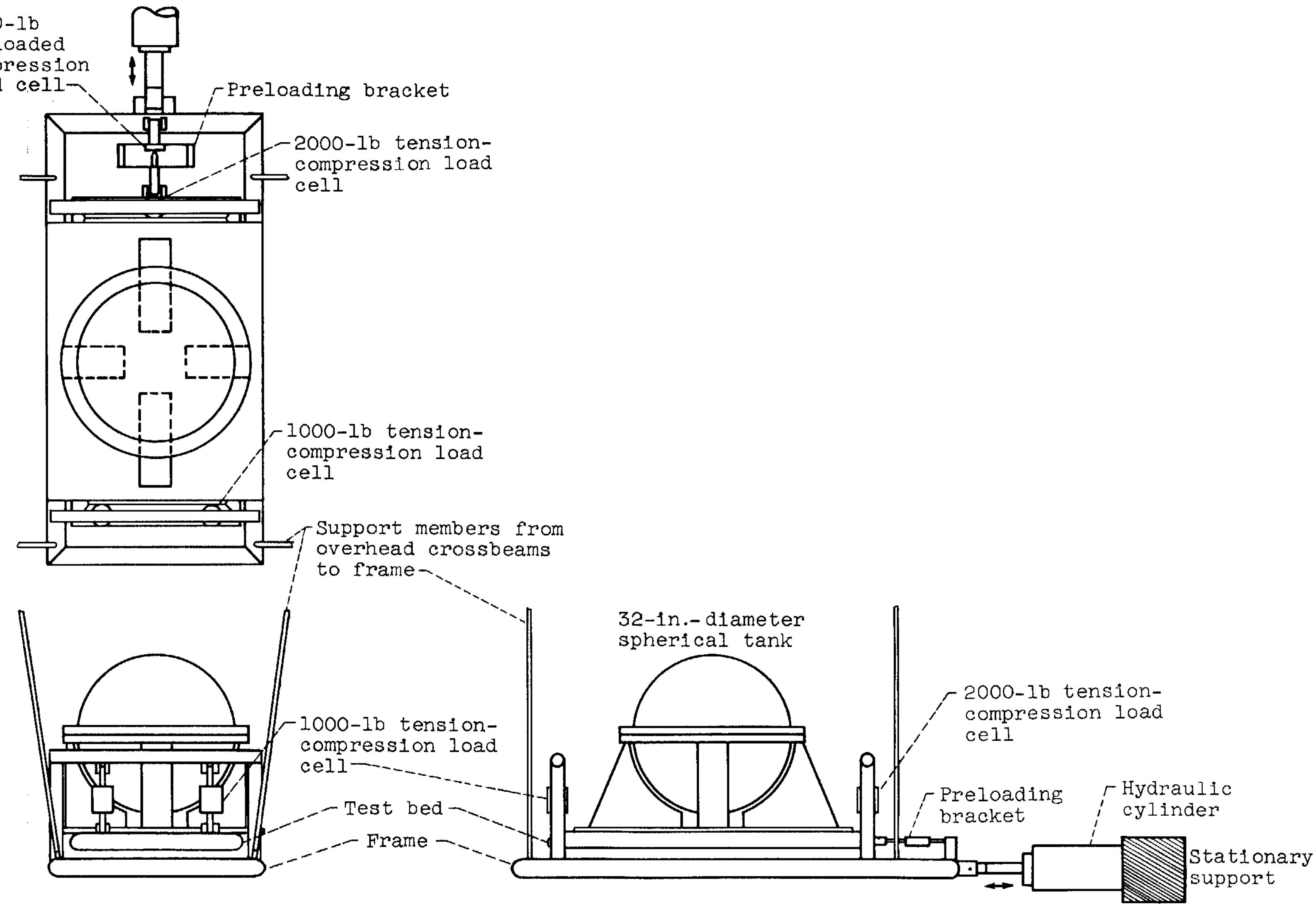
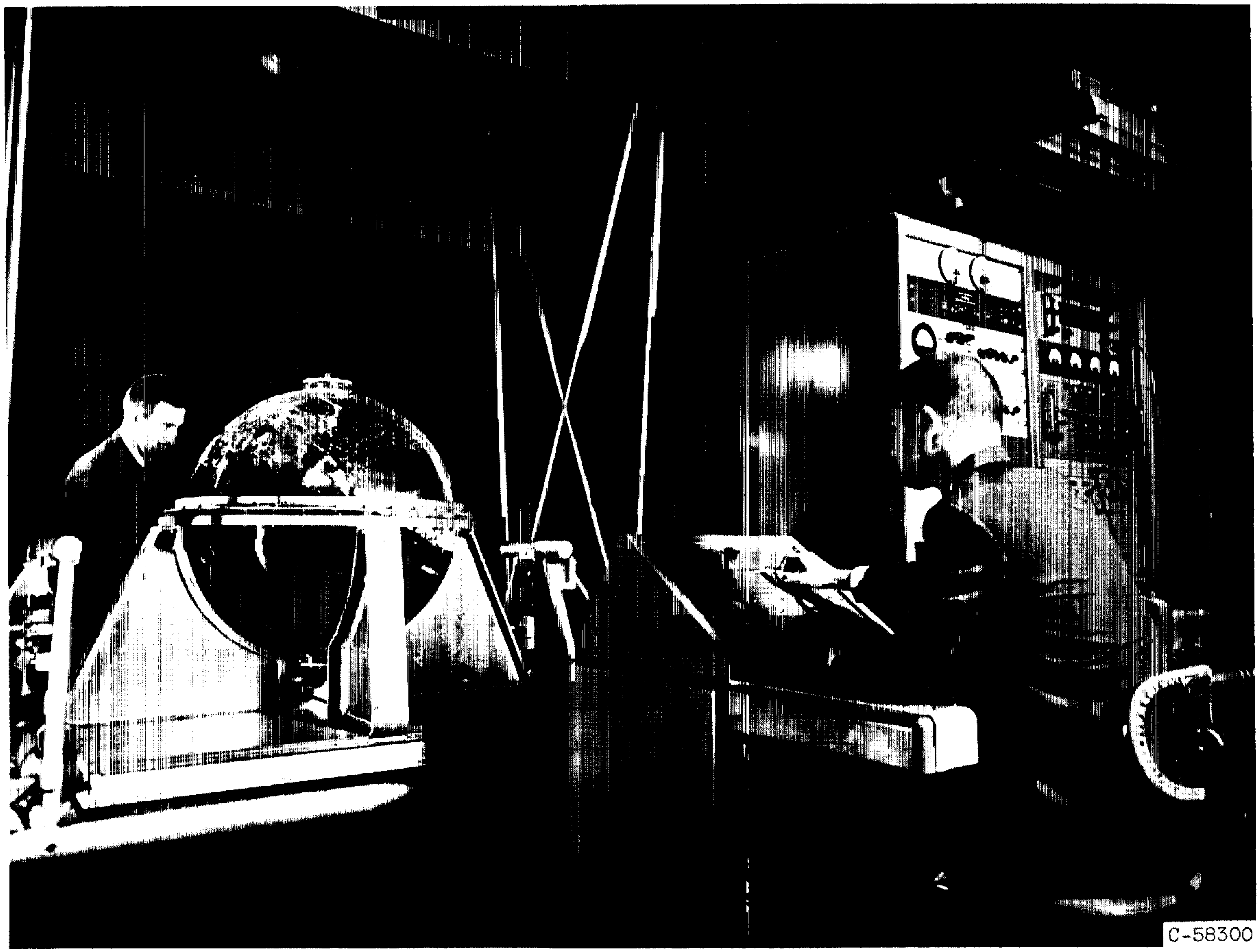


Figure 1. - Small experimental slosh force test facility.



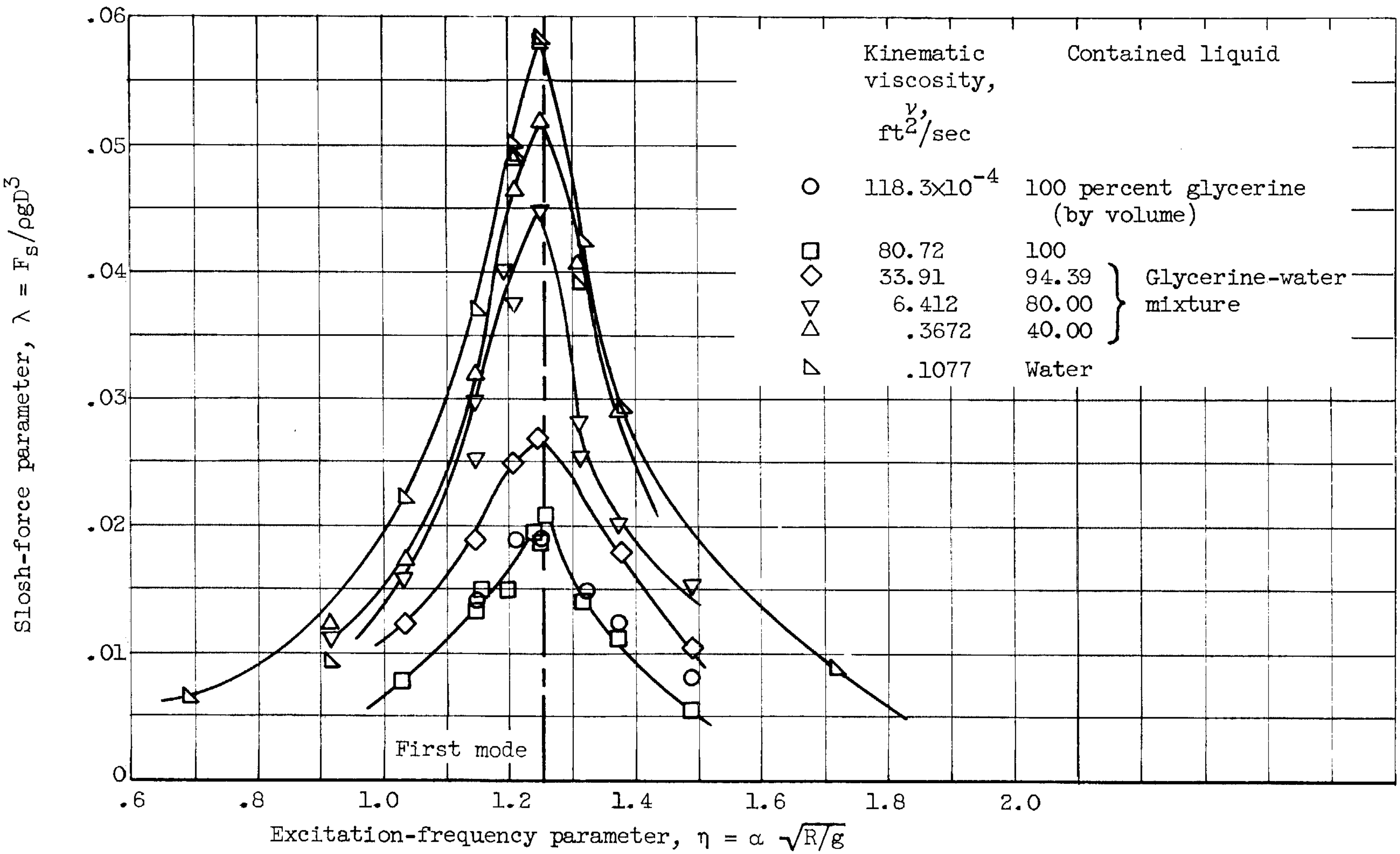
(a) Schematic view.

Figure 2. - Large experimental slosh force test facility.



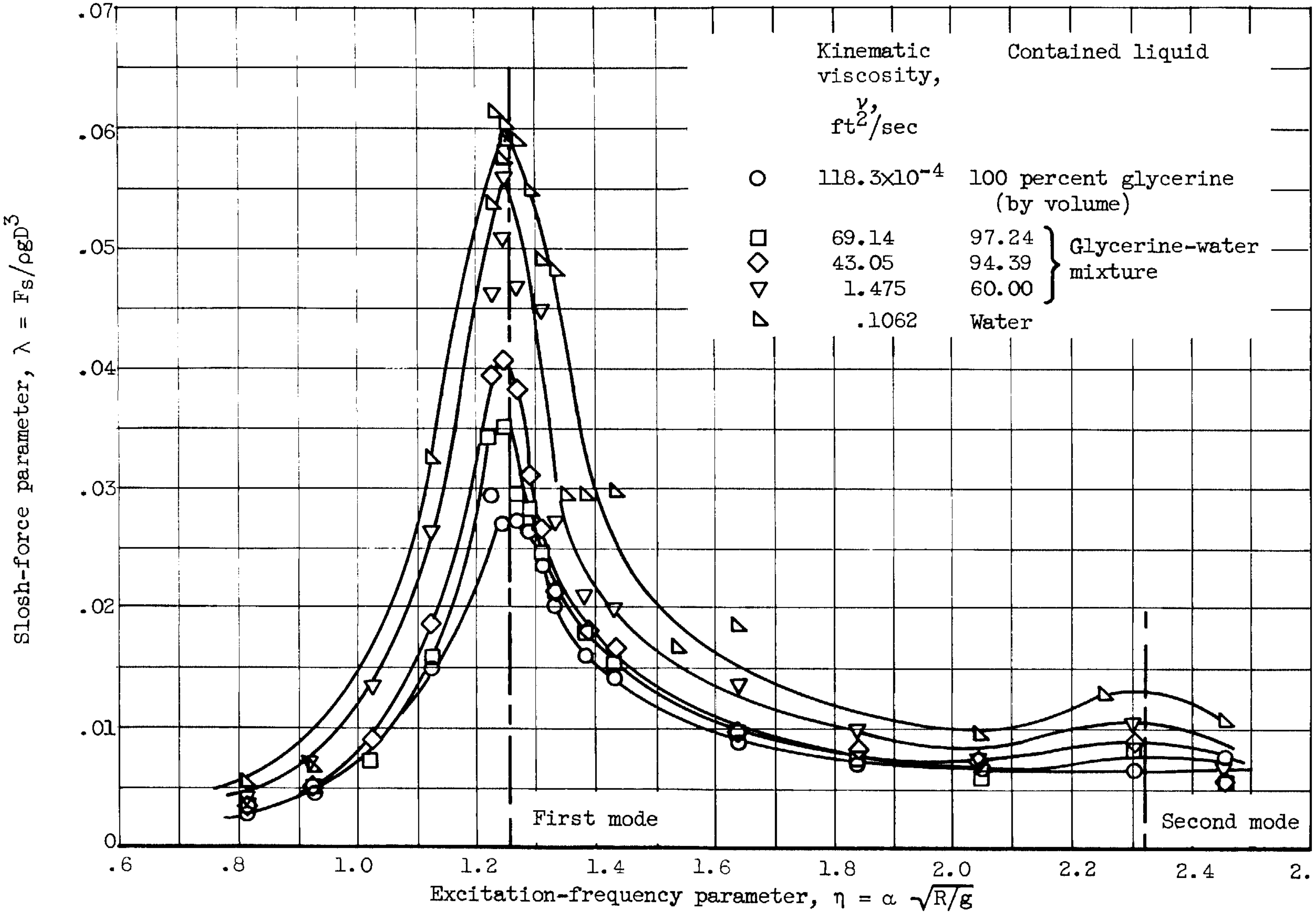
(b) Overall view.

Figure 2. - Concluded. Large experimental slosh force test facility.



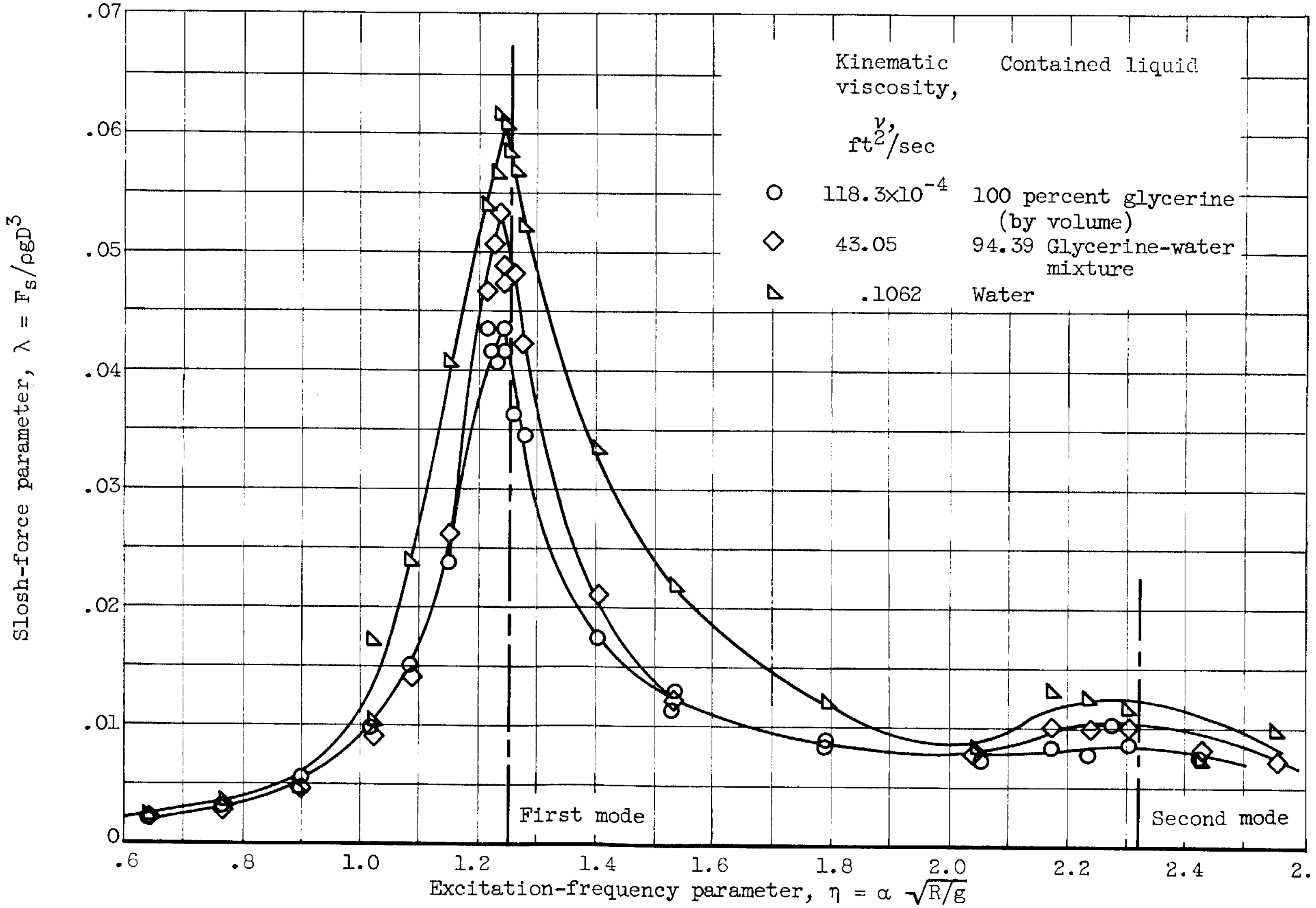
(a) Spherical tank diameter, 9.5 inches; excitation-amplitude parameter, $X_o/D = 0.0105$; excitation amplitude, $X_o = 0.100$ inch.

Figure 3. - Effect of kinematic viscosity on slosh-force parameter.



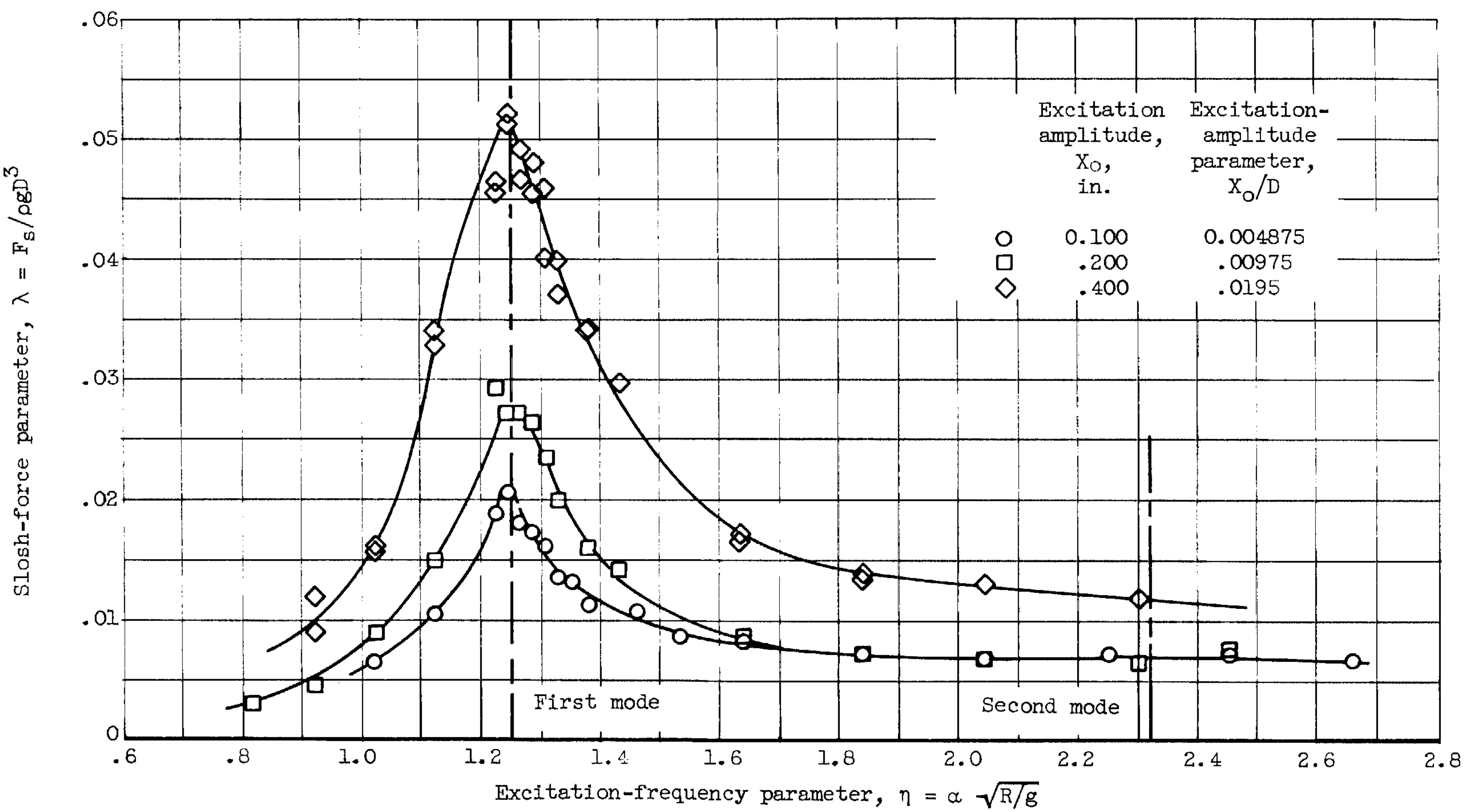
(b) Spherical tank diameter, 20.5 inches; excitation-amplitude parameter, $X_o/D = 0.00975$; excitation amplitude, $X_o = 0.200$ inch.

Figure 3. - Continued. Effect of kinematic viscosity on slosh-force parameter.



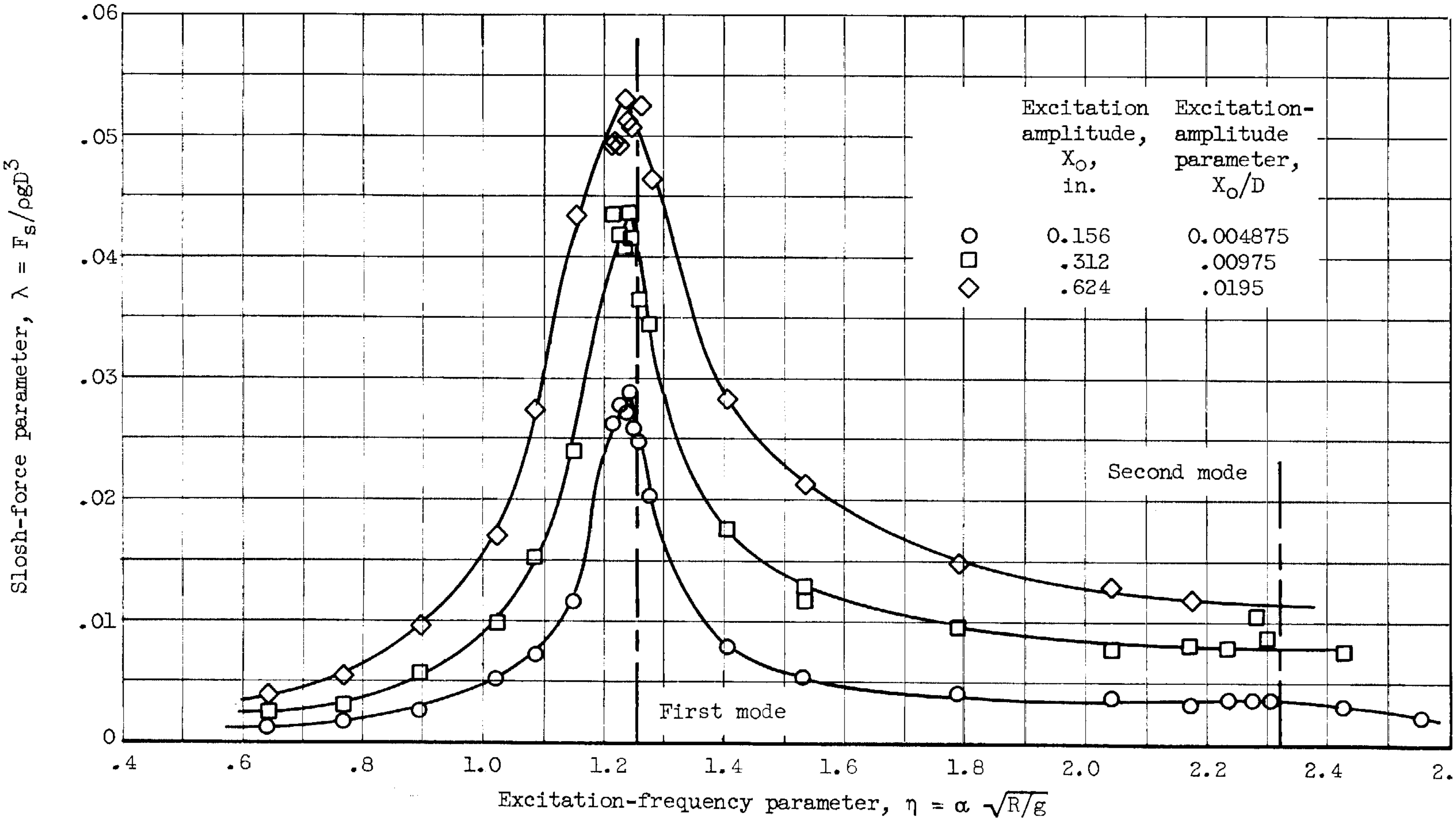
(c) Spherical tank diameter, 32.0 inches; excitation-amplitude parameter, $X_o/D = 0.00975$; excitation amplitude, $X_o = 0.312$ inch.

Figure 3. - Concluded. Effect of kinematic viscosity on slosh-force parameter.



(a) Spherical tank diameter, 20.5 inches.

Figure 4. - Effect of excitation-amplitude parameter on slosh-force parameter using 100 percent glycerine (kinematic viscosity, $\nu = 118.3\times10^{-4}$ ft^2/sec).



(b) Spherical tank diameter, 32.0 inches.

Figure 4. - Concluded. Effect of excitation-amplitude parameter on slosh-force parameter using 100 percent glycerine (kinematic viscosity, $\nu = 118.3\times10^{-4}$ ft^2/sec).

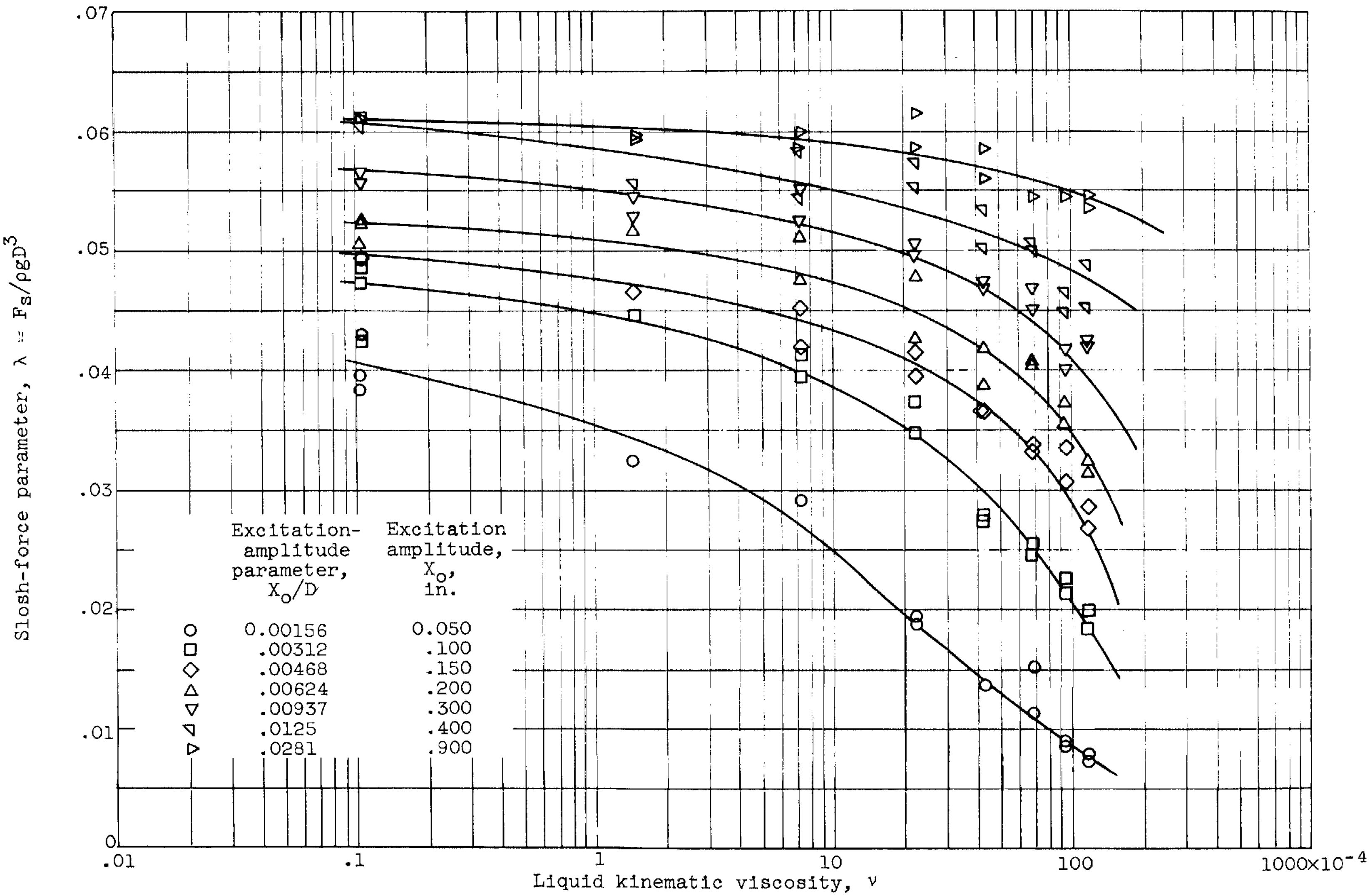


Figure 5. - Effect of liquid kinematic viscosity on first mode slosh-force parameter for 32.0-inch-diameter spherical tank.

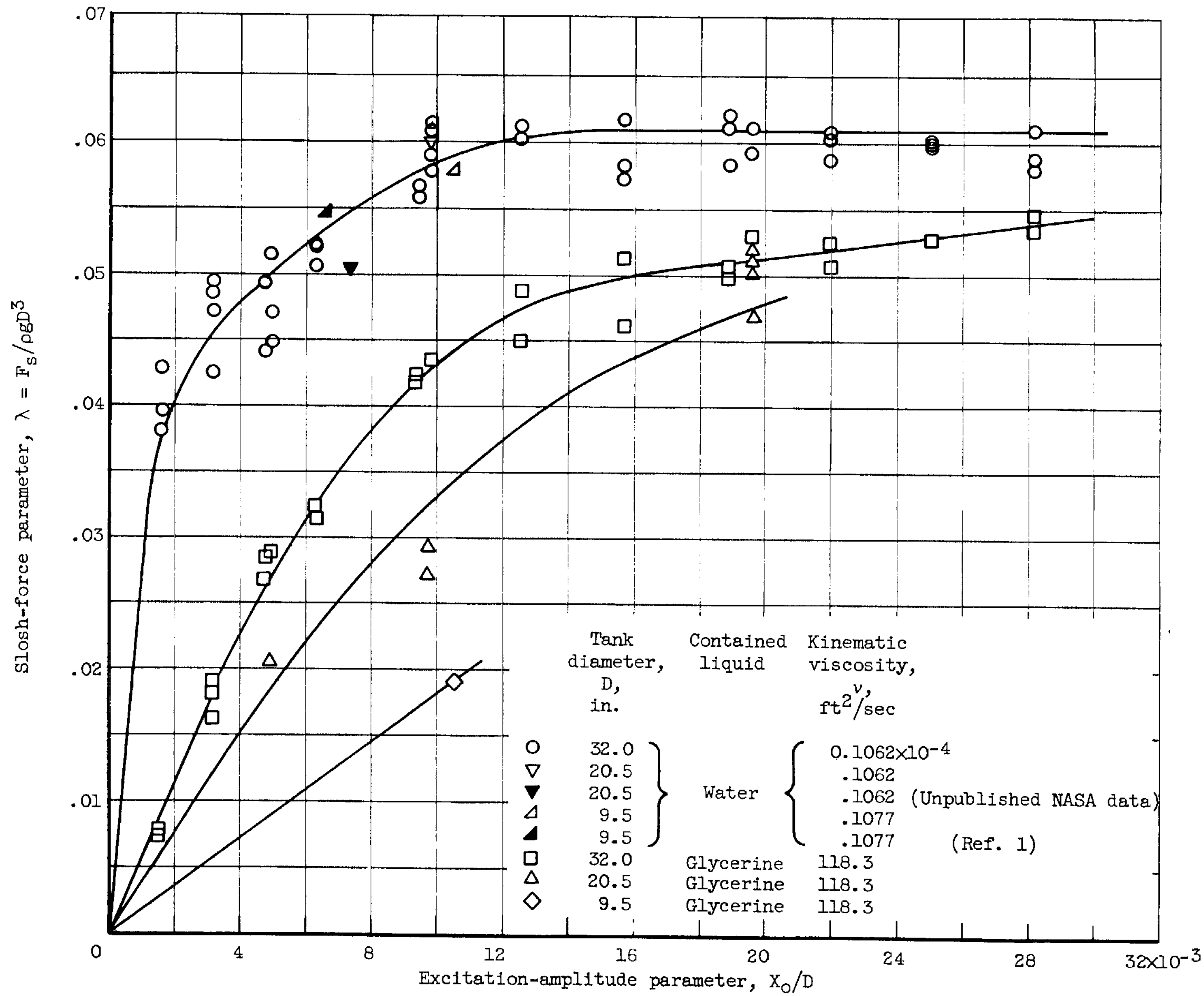


Figure 6. - First mode slosh-force parameter as a function of excitation-amplitude parameter for water and glycerine.

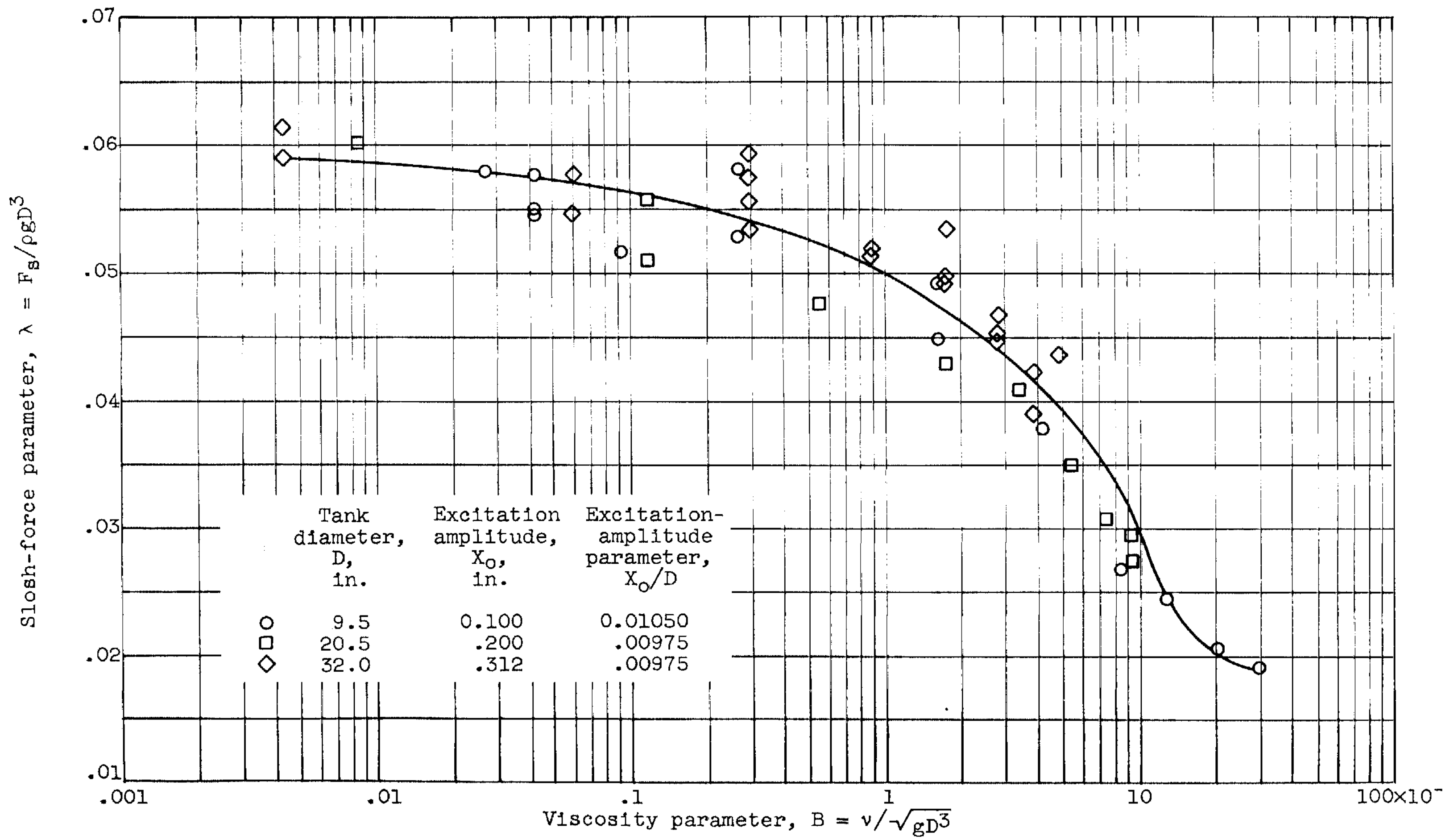


Figure 7. - First mode slosh-force parameter as a function of viscosity parameter.

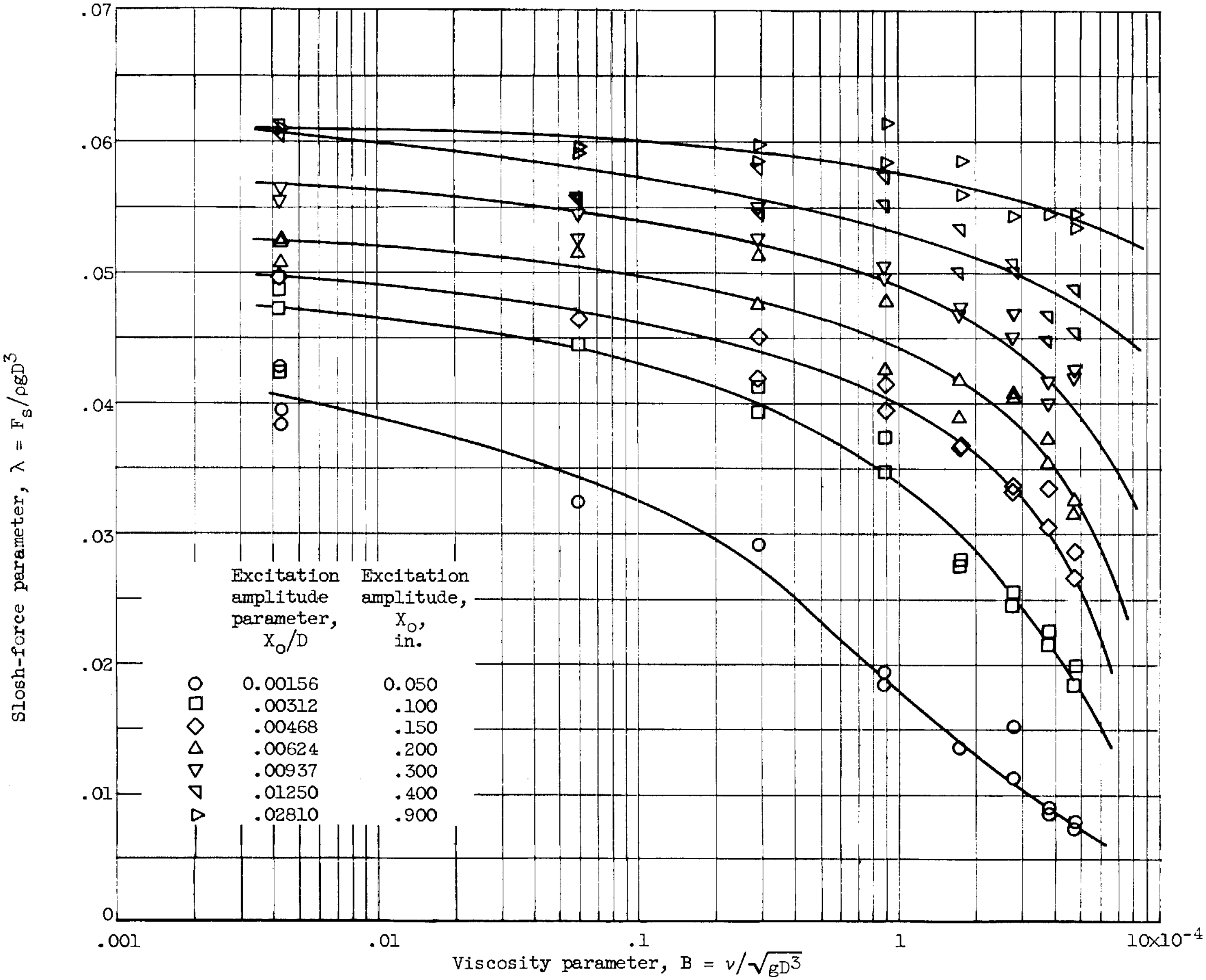
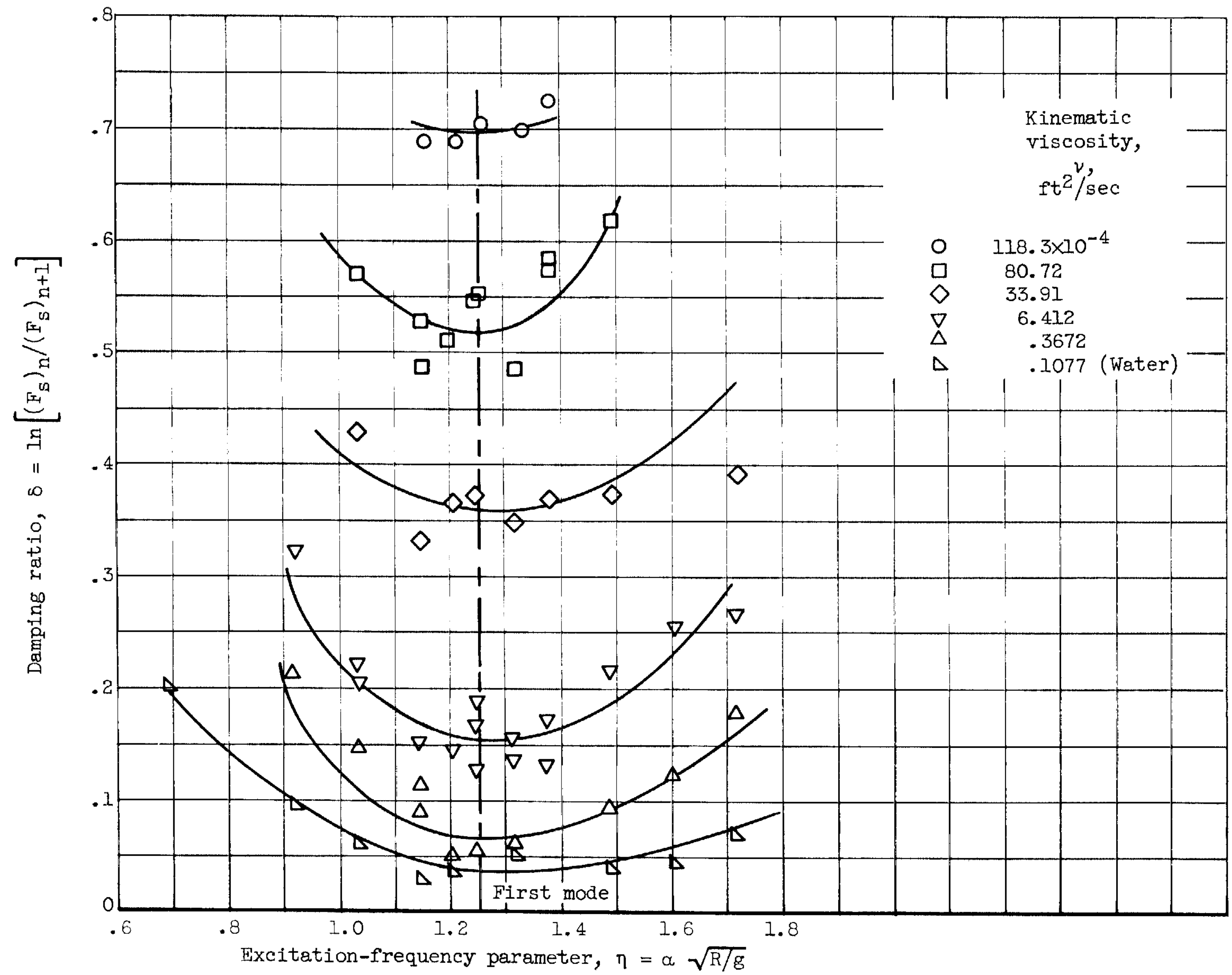
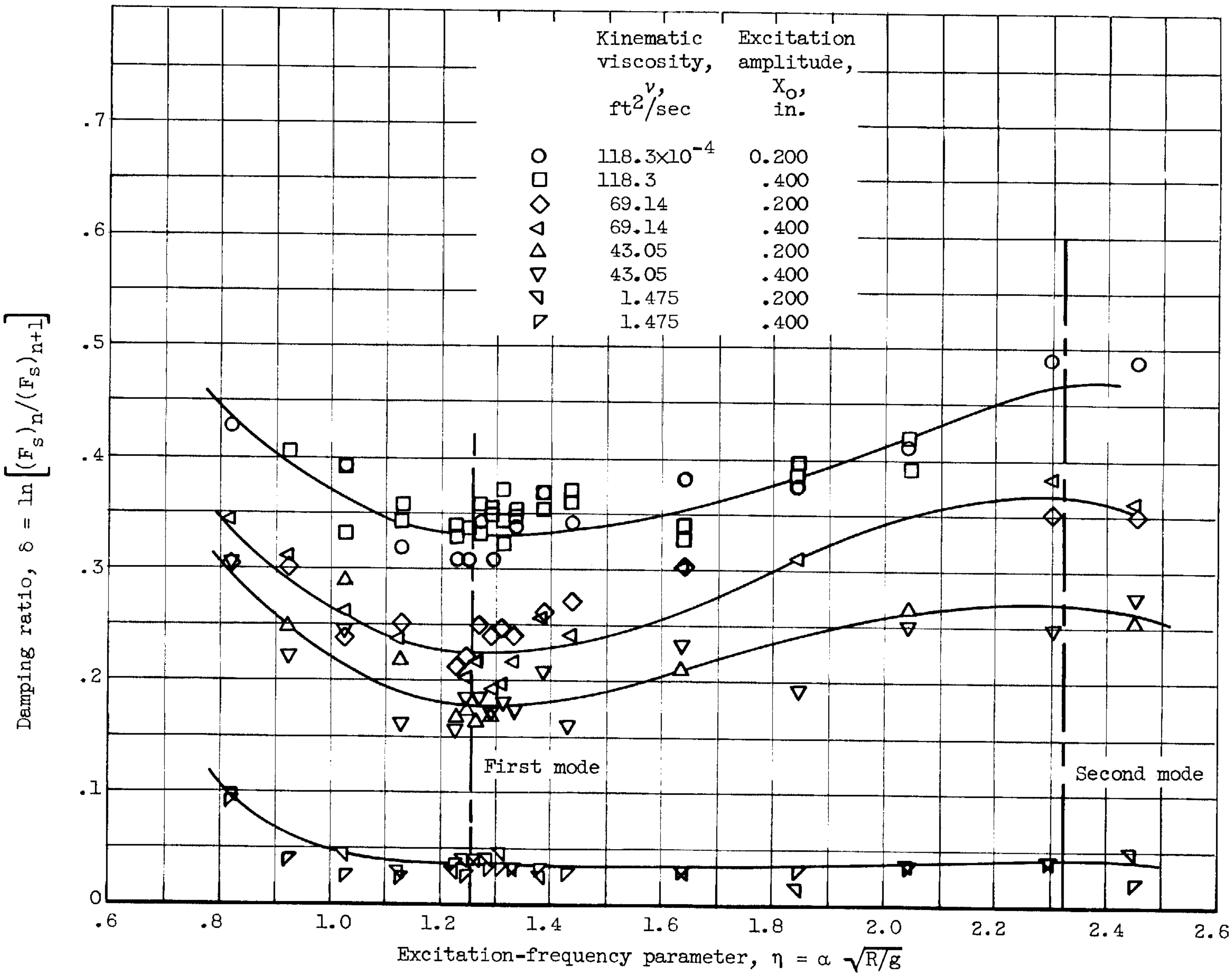


Figure 8. - First mode slosh-force parameter as a function of viscosity parameter for 32.0-inch-diameter spherical tank.



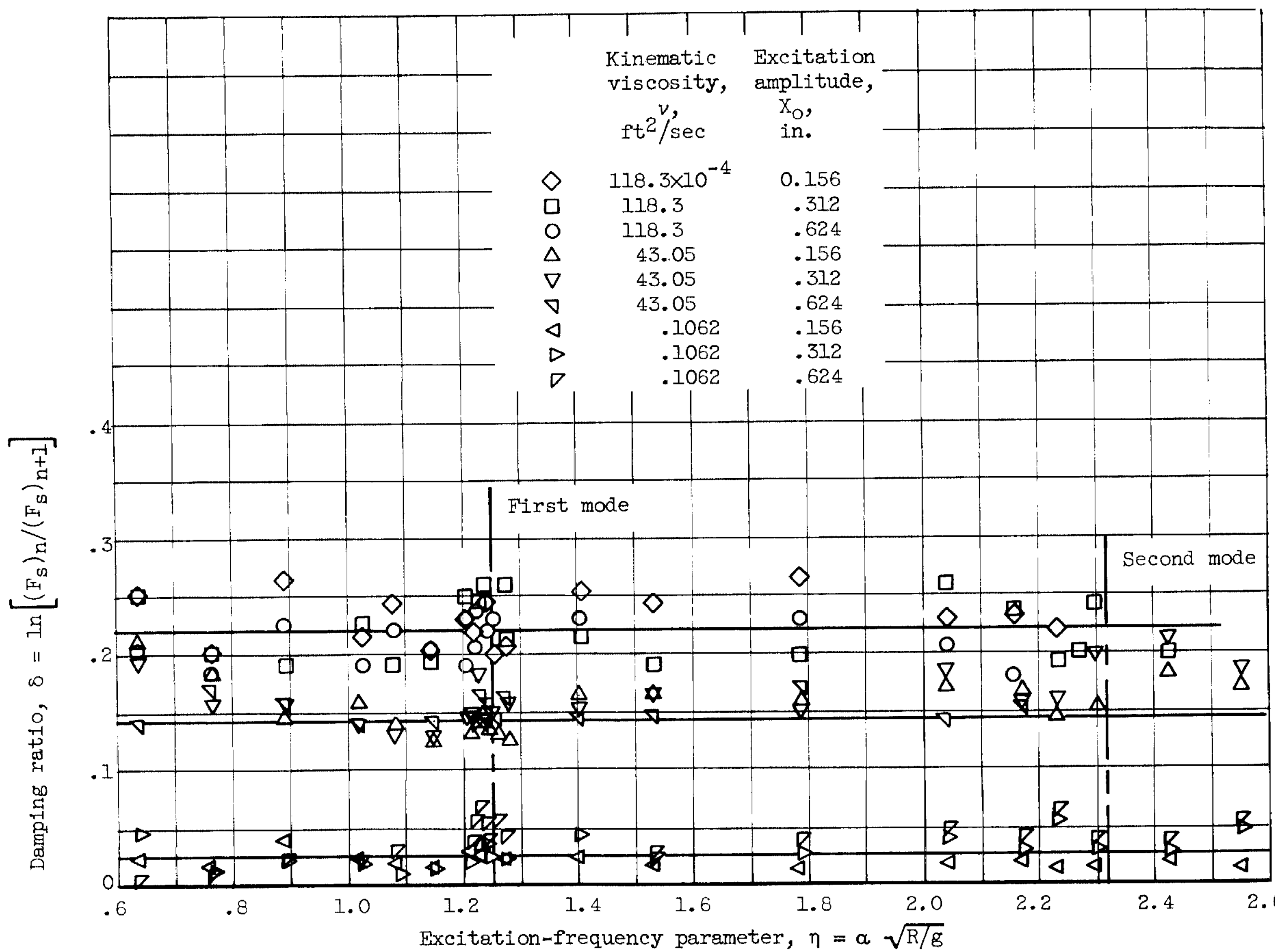
(a) Spherical diameter, 9.5 inches; excitation-amplitude parameter, $X_o/D = 0.0105$.

Figure 9. - Effect of kinematic viscosity on damping ratio.



(b) Spherical tank diameter, 20.5 inches.

Figure 9. - Continued. Effect of kinematic viscosity on damping ratio.



(c) Spherical tank diameter, 32.0 inches.

Figure 9. - Concluded. Effect of kinematic viscosity on damping ratio.

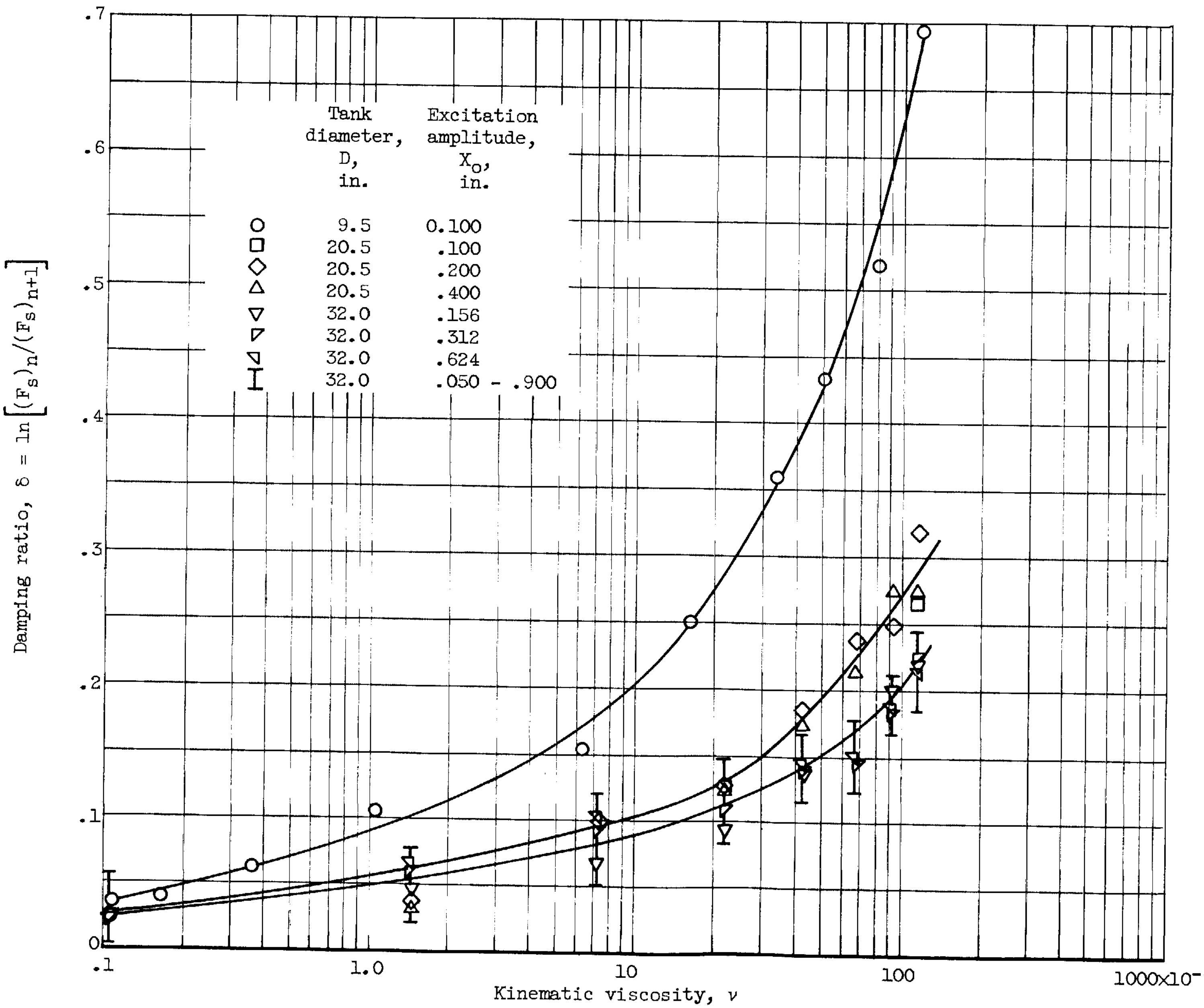


Figure 10. - Average first mode damping ratio as a function of liquid kinematic viscosity.

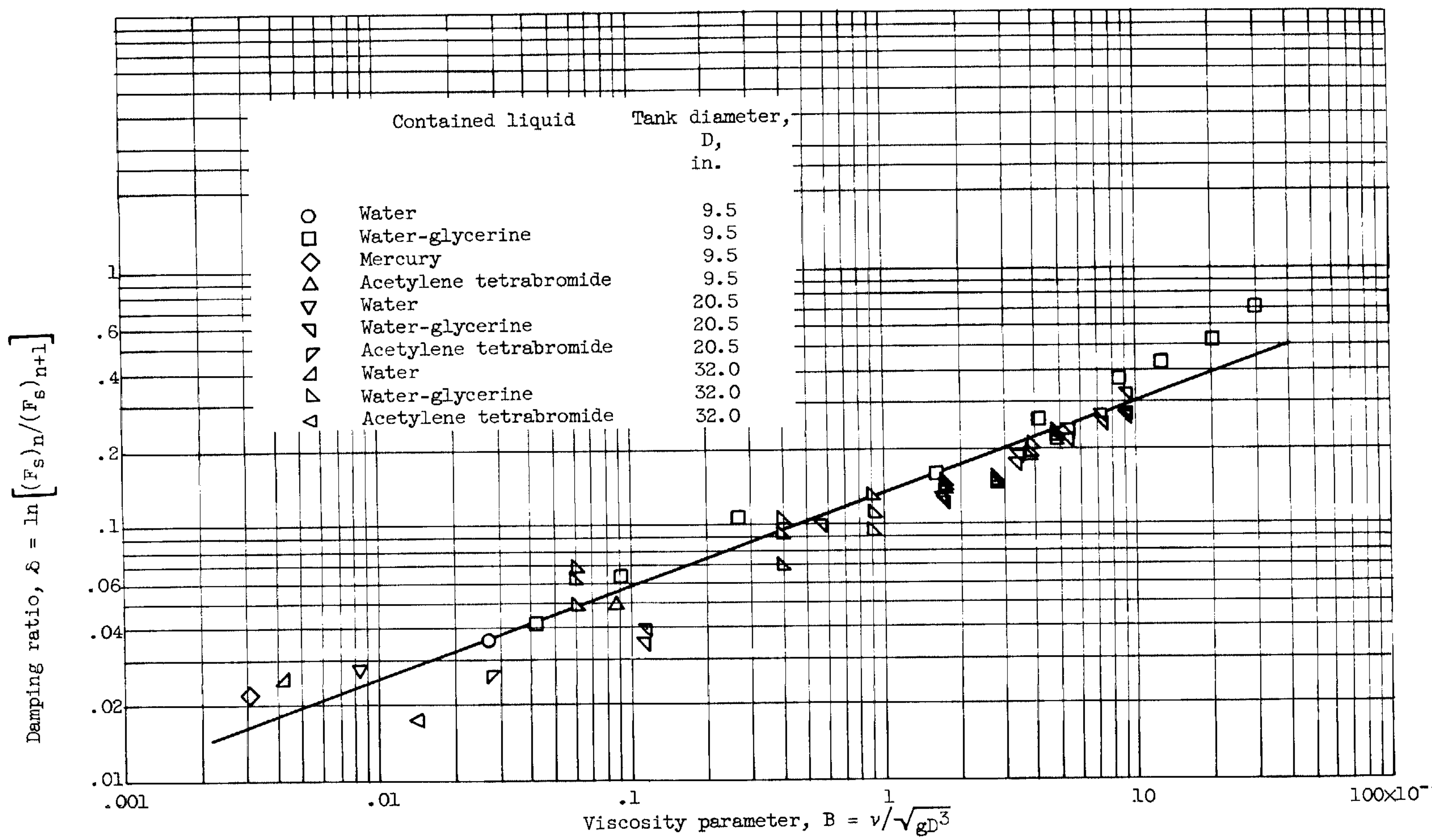


Figure 11. - Average first mode damping ratio as a function of viscosity parameter.

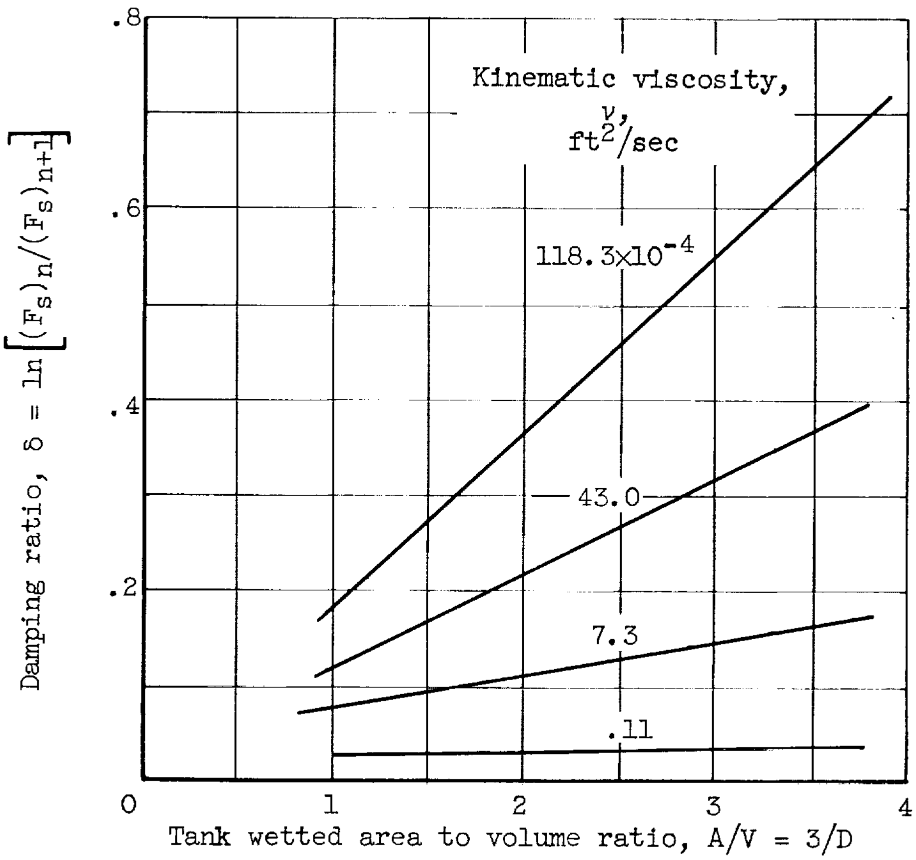


Figure 12. - Effect of the A/V ratio on the average first mode damping ratio for varying viscosity liquids, where $0.050 \leq X_o \leq 0.900$ inch.